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Existence and nonexistence of global solutions to the Cauchy problem of a class of nonlocal convective reaction–diffusion equations

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Abstract

We study the existence and nonexistence of global solutions to the Cauchy problem

$$u_t - \Delta u = \left(\int_{R^N} |u(t,x)|^\sigma \, \mathrm{d}x\right)^{p/\sigma} |u|^{r-1}u + \boldsymbol{a}\cdot \bigtriangledown (|u|^{q-1}u) \qquad t>0, \quad x\in R^N$$
$$u(0,x) = u_0(x) \qquad x\in R^N$$

where $u(t,x)$ is a scalar function, $\boldsymbol{a} \in R^N, \boldsymbol{a} \neq 0, p \geqslant 0, q, \sigma, r \geqslant 1$. $\bigtriangledown$ is a gradient operator. The results obtained generalize the results of Aguirre and Escobedo (J Aguirre and M Escobedo 1993 *Proc. R. Soc. Edin.* A **123** 433–60), which do not consider the nonlocal factor in the reaction term of the equation, and also generalize the results of Wang *et al* (M X Wang, S Wang and C H Xie 1999 *J. Partial Diff. Eqs.* **12** 201–11) which do not include the nonlinear convection term in the equation.

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1. Introduction

We study the existence and nonexistence of global solutions to the Cauchy problem

$$\begin{cases} u_t - \Delta u = \left(\int_{R^N} |u(t,x)|^\sigma \, \mathrm{d}x\right)^{p/\sigma} |u|^{r-1}u + \boldsymbol{a}\cdot \bigtriangledown (|u|^{q-1}u) & t>0, \quad x\in R^N \\ u(0,x) = u_0(x) & x\in R^N \end{cases} \tag{1}$$

where $u(t,x)$ is a scalar function, $\boldsymbol{a} \in R^N, \boldsymbol{a} \neq 0, p \geqslant 0, q, \sigma, r \geqslant 1$. $\bigtriangledown$ is a gradient operator. More precisely, given $u_0(x) \in L^s(R^N)(1 \leqslant s \leqslant \infty)$, let $T_{\max} > 0$ be the maximal time of existence of the solution to problem (1). Then, as we shall prove in section 2, either $T_{\max} = \infty$ and the solution is said to be global, or $T_{\max} < \infty$ and then

$$\lim_{t\to T_{\max}^-} \left(\|u\|_{L^s(R^N)} + \|u\|_{L^\sigma(R^N)}\right) = \infty. \tag{2}$$

In the latter case we say the solution of (1) blows up in $L^s(R^N) \cup L^\sigma(R^N)$. Our aim is to discuss which of these two possibilities occurs in terms of p, q, r, σ, N and $u_0(x)$.

The corresponding problem for the reaction–diffusion equation

$$\begin{cases} u_t - \Delta u = |u|^{p-1}u & t > 0, \quad x \in R^N \\ u(0, x) = u_0(x) & x \in R^N \end{cases} \tag{3}$$

is by now fairly well understood. The classical results of Fujita [1], Bandle and Levine [2] and Weissler [3] state:

(1) if $1 < p < 1 + 2/N$, then all positive solutions of (3) blow up in finite time;
(2) if $p > 1 + 2/N$, then global positive solutions of (3) exist if the initial value is small enough, and blow-up in finite time occurs if it is sufficiently large.

In the critical case $p = 1 + 2/N$, all positive solutions blow up in finite time [4–7].

Aguirre and Escobedo [8] studied the effect of the nonlinear convection term $\boldsymbol{a} \cdot \nabla (u^q)$ on the global existence and blow-up of solutions. More precisely, they considered the following Cauchy problem of generalized Burgers-type convective reaction–diffusion equation:

$$\begin{cases} u_t - \Delta u = |u|^{p-1}u + \boldsymbol{a} \cdot \nabla (|u|^{q-1}u) & t > 0, \quad x \in R^N \\ u(0, x) = u_0(x) & x \in R^N \end{cases} \tag{4}$$

where $\boldsymbol{a} \in R^N$, $\boldsymbol{a} \neq 0$. They obtained the following results:

Theorem A. *Let $p > 1$ and $q \geqslant 1$ be given.*

(1) If $q = 1$, then provided $1 < p \leqslant 1 + 2/N$ all positive solutions of (4) blow up in finite time, while if $p > 1 + 2/N$, both global and blowing up solutions exist;
(2) If $1 < q \leqslant p \leqslant \min\{1 + 2/N, 1 + 2q/(N + 1)\}$, then all positive solutions of (4) blow up in finite time;
(3) If $q > 1$ and $p > \min\{1 + 2/N, 1 + 2q/(N + 1)\}$, then there exist global positive solutions of (4). More precisely, there exists a constant C such that if $\|u_0\|_1 + \|u_0\|_\infty \leqslant C$, then the solution of (4) is global;
(4) If $q \leqslant p$ and $p > \min\{1 + 2/N, 1 + 2q/(N + 1)\}$, then the solution of (4) with sufficiently large initial value $u_0(x) \geqslant 0$ blows up in finite time.

Let us explain what is meant by 'sufficiently large' in part (4) of theorem A. We fix a positive function $\phi \in C^2(R^N) \cap L^1(R^N)$ such that

$$\int_{R^N} \phi(x)\,\mathrm{d}x = 1 \qquad \Delta\phi(x) \geqslant -\phi(x) \qquad |\nabla \phi(x)| \leqslant K\phi(x) \tag{5}$$

for some constant $K > 0$. Examples of such functions are

$$\phi(x) = C \exp\left(-\frac{\delta}{N}\sqrt{\delta^2 + |x|^2}\right)$$

$$\phi(x) = C(2N\delta + |x|^2)^{-\gamma} \qquad \gamma > \frac{N}{2}$$

for any $\delta > 0$ and the appropriate constant $C > 0$. A sufficient condition for the solution of (4) to blow up is for some $\lambda > 0$ ($0 < \lambda < 1/(2K|\boldsymbol{a}|)$ if $p = q$)

$$\int_{R^N} u_0(x)\phi(\lambda x)\,\mathrm{d}x > \begin{cases} \max\left\{2^{1/(p-1)}\lambda^{2/(p-1)-N}, (2K|\boldsymbol{a}|)^{1/(p-q)}\lambda^{1/(p-q)-N}\right\} & q < p \\ 2^{1/(p-1)}\lambda^{2/(p-1)-N} & q = p. \end{cases} \tag{6}$$

Another generalization for problem (3) is to consider the case of the equation including the nonlocal factor $\left(\int_{R^N} |u(t,x)|^\sigma \,\mathrm{d}x\right)^{p/\sigma}$ in a nonlinear reaction term. Many physics phenomena can be described by nonlocal mathematical models, and a few authors have studied it, for example, [9–13] and references cited therein. Recently, Wang *et al* [13] proved the existence of a critical exponent of a Fujita-type for the Cauchy problem of a class of nonlocal reaction–diffusion system. For our requirement, consider the following problem:

$$\begin{cases} u_t - \Delta u = \left(\int_{R^N} u^\sigma(t,x)\,\mathrm{d}x\right)^{p/\sigma} u^r & t > 0, \quad x \in R^N \\ u(0,x) = u_0(x) & x \in R^N \end{cases} \tag{7}$$

where $p \geqslant 0, \sigma, r \geqslant 1, p+r > 1, u_0(x) \geqslant 0$ and $u_0(x) \in L^\sigma(R^N) \cap L^\infty(R^N)$. We can easily deduce the results from [13]:

Theorem B.

(1) If $1 < p + r \leqslant 1 + 2/N + p/\sigma$, then all positive solutions of (7) blow up in finite time;
(2) If $p + r > 1 + 2/N + p/\sigma$, then solutions of (7) blow up in finite time for sufficiently large $u_0(x) > 0$, while global solutions exist for sufficiently small $u_0(x) > 0$.

The most common interpretation of (1) is to think of u as the temperature of a substance in R^N subject to a chemical reaction. Bebernes and Bressan [10] studied an ignition model for a compressible reactive gas which is a nonlocal reaction–diffusion equation. We take the convection effects into consideration and get model (1). So, the aim of this paper is to study the effect of the nonlinear convection term $\boldsymbol{a} \cdot \nabla(u^q)$, added into the right side of the equation in problem (7), on the global existence and blow-up of solutions. The effect will be seen by comparing the results of theorem B with ours. In another way, based on problem (4), we study whether (1) has results similar to [8] when the nonlinear term $|u|^{p-1}u$ in (4) is replaced by $\left(\int_{R^N} |u(t,x)|^\sigma \,\mathrm{d}x\right)^{p/\sigma} |u|^{r-1}u$.

In problem (1), $q = 1$ is a special case. Any solution u in that case can be written as

$$u(t,x) = v(t, x + t\boldsymbol{a})$$

where v is the solution of (7). In this case we see that if v blows up, then so does u and vice versa. Therefore, the convection term $\boldsymbol{a} \cdot \nabla u$ has no effect on whether solutions are global or blow up in finite time. We will see that this is not true for all values of q.

We now state our main results in the following theorem:

Theorem 1. *Let $p \geqslant 0$, q, σ and $r \geqslant 1$ be given.*

(1) If $q = 1$, then provided $1 < p + r \leqslant 1 + 2/N + p/\sigma$, all positive solutions of (1) blow up in finite time, while, if $p + r > 1 + 2/N + p/\sigma$, both global and blowing up solutions exist;
(2) If $q > 1$ and

$$p + r > 1 + \frac{p}{\sigma} + \frac{2}{N} + \min\left\{0, \frac{(Nq - N - 1)(r-1)}{Nq}\right\}$$

then there exists a constant $C > 0$ such that when the non-negative function $u_0 \in L^1 \cap L^\infty \cap L^\sigma$ satisfies

$$\|u_0\|_1 + \|u_0\|_\infty + \|u_0\|_\sigma \leqslant C$$

problem (1) has a non-negative global solution.

(3) *Let* $1 < q \leqslant \sigma(p+r)/(p+\sigma)$. *If for some* $\lambda > 0$ $(0 < \lambda < 1/(2K|\boldsymbol{a}|)$ *if* $p = 0, 1 < q = r)$

$$\int_{R^N} u_0(x)\phi(\lambda x)\,\mathrm{d}x > \begin{cases} \max\left\{2^{\frac{1}{p+r-1}}\lambda^{\frac{2}{p+r-1}-N}, (2K|\boldsymbol{a}|)^{\frac{1}{p+r-q}}\lambda^{\frac{1}{p+r-q}-N}\right\} \\ \qquad p > 0 \quad 1 < q \leqslant \frac{\sigma(p+r)}{p+\sigma} \\ \max\left\{2^{\frac{1}{r-1}}\lambda^{\frac{2}{r-1}-N}, (2K|\boldsymbol{a}|)^{\frac{1}{r-q}}\lambda^{\frac{1}{r-q}-N}\right\} \\ \qquad p = 0 \quad 1 < q < r \\ 2^{\frac{1}{r-1}}\lambda^{\frac{2}{r-1}-N} \qquad p = 0 \quad 1 < q = r \end{cases}$$

then the non-negative solution u of (1) blows up in finite time, where the function $\phi(x)$ *is given by (5).*

We give the plan of our paper: in section 2, we prove the existence of local solutions of (1); in section 3, we discuss the existence of global solutions and in section 4, we study the blow-up conditions.

2. Local solutions

In this section we prove the existence and uniqueness of the local solution of (1) when the initial function $u_0(x)$ is given in $L^s(R^N) \cap L^\sigma(R^N)$, where $1 \leqslant s \leqslant \infty$. Let us now introduce some notation. Given a function u defined on $(0, T) \times R^N$, we denote the function $u(t, \cdot)$ and its $L^m(R^N)$ norm by $u(t)$ and $\|u(t)\|_m$, respectively, and define

$$\Psi_1(u)(t) = \int_0^t K(t-s) * \left(\|u(s)\|_\sigma^p |u(s)|^{r-1} u(s)\right)\,\mathrm{d}s$$

$$\Psi_2(u)(t) = \int_0^t \boldsymbol{a} \cdot \bigtriangledown K(t-s) * (|u(s)|^{q-1} u(s))\,\mathrm{d}s$$

$$\Psi(u) = \Psi_1(u)(t) + \Psi_2(u)(t)$$

where $K(t) = (4\pi t)^{-N/2}\exp(-|x|^2/4t)$ is the heat kernel, and $*$ is convolution.

We first prove the existence of solutions of the corresponding integral equation

$$u(t) = K(t) * u_0 + \Psi(u) = \Phi(u). \tag{8}$$

Then by argument of regularity and uniqueness of the solution, we have

Theorem 2. *Let* $1 \leqslant s \leqslant \infty$ *and* $u_0 \in L^s(R^N) \cap L^\sigma(R^N)$ *be given.*

(1) *If* $1 \leqslant s < \infty, 1 \leqslant r < 1 + 2\min\{s, \sigma\}/N$ *and* $1 \leqslant q \leqslant 1 + \min\{s, \sigma\}/N$ *or* $1 < s < \infty, r = 1 + 2\min\{s, \sigma\}/N$ *and* $1 \leqslant q \leqslant 1 + \min\{s, \sigma\}/N$, *then there exist a* $T > 0$ *and a unique classical solution u of (1) in* $(0, T) \times R^N$ *such that*

$$\|u(t)\|_\sigma,\ \|u(t)\|_s,\ t^{\frac{N}{2s}(1-\frac{1}{r})}\|u(t)\|_{sr},\ t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|u(t)\|_{sq}$$

are bounded in $(0, T)$ *and* $u(t)$ *converges to* u_0 *in the* L^s*-norm as* $t \to 0^+$; *if* $s = \infty$, *for any* $p \geqslant 0, r, \sigma \geqslant 1, p + r > 1$, *there is a* $T > 0$ *and a unique classical solution u of (1) in* $(0, T) \times R^N$ *such that* $u(t)$ *converges almost everywhere to* u_0 *as* $t \to 0^+$.

(2) *Fix s satisfying the conditions in (1). Then either the solution u exists for all time* $t > 0$ *in* $L^s(R^N) \cap L^\sigma(R^N)$ *or there exists a maximal time of existence* $0 < T_{\max} < \infty$ *such that*

$$\lim_{t \to T_{\max}^-} (\|u(t)\|_s + \|u(t)\|_\sigma) = \infty$$

(3) *If* u_0 *is non-negative, then so is the solution u.*

Proof. We give only the partial proof of the theorem, stressing the difference between the proof of theorem 2.1 of [8] and ours.

(1) Consider the case $1 \leqslant s < \infty$. Choose $R > 0$ such that for all $t > 0$

$$\|K(t) * u_0\|_\sigma < R \qquad \|K(t) * u_0\|_\sigma < R$$

$$t^{\frac{N}{2s}(1-\frac{1}{r})}\|K(t) * u_0\|_{sr} < R \qquad t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|K(t) * u_0\|_{sq} < R.$$

Since

$$\|K(t) * u_0\|_\sigma \leqslant \|u_0\|_\sigma \qquad \|K(t) * u_0\|_s \leqslant \|u_0\|_s$$

and

$$t^{\frac{N}{2s}(1-\frac{1}{m})}\|K(t) * u_0\|_{sm} < C\|u_0\|_s$$

for $m = r$ or q, we can take R as a positive constant multiple of $\max\{\|u_0\|_\sigma, \|u_0\|_s\}$.

First suppose that $r < 1 + 2\min\{s,\sigma\}/N$, $q < 1 + \min\{s,\sigma\}/N$. Given $T > 0$, let

$$E = \left\{u : [0,T) \times R^N \to R : \|u(t)\|_\sigma \leqslant 2R, \|u(t)\|_s \leqslant 2R,\right.$$
$$\left. t^{\frac{N}{2s}(1-\frac{1}{r})}\|u(t)\|_{sr} \leqslant 2R, t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|u(t)\|_{sq} \leqslant 2R, \forall t \in (0,T)\right\}.$$

E is a complete metric space for the distance defined by the above expressions. If $u, v \in E$, it easily follows that

$$\|\Phi(u)\|_\sigma, \quad \|\Phi(u)\|_s, \quad t^{\frac{N}{2s}(1-\frac{1}{r})}\|\Phi(u)\|_{sr}, \quad t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|\Phi(u)\|_{sq} \tag{9}$$

are bounded by

$$R + C\left(R^{p+r}T^{1-\frac{N(r-1)}{2\min\{\sigma,s\}}} + |a|R^q T^{\frac{1}{2}-\frac{N(q-1)}{2\min\{\sigma,s\}}}\right)$$

for some constant $C > 0$. Similarly, the quantities $\Phi(u)$ in (9) evaluated for the difference $\Phi(u) - \Phi(v)$ are bounded by a constant times

$$R^{p+r-1}T^{1-\frac{N(r-1)}{2\min\{\sigma,s\}}} \sup_{0<t<T} t^{\frac{N}{2s}(1-\frac{1}{r})}\|u - v\|_{sr} + |a|R^{q-1}T^{\frac{1}{2}-\frac{N(q-1)}{2\min\{\sigma,s\}}} \sup_{0<t<T} t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|u - v\|_{sq}.$$

We then take $T > 0$ small enough so that Φ is a contraction on E, and therefore it has a fixed point u which is a mild solution of (1).

Now suppose $r < 1 + 2\min\{s,\sigma\}/N$, $q = 1 + \min\{s,\sigma\}/N$. For $T, b > 0$, we define

$$E = \left\{u : [0,T) \times R^N \to R : \|u(t)\|_\sigma \leqslant 2R, \|u(t)\|_s \leqslant 2R, t^{\frac{N}{2s}(1-\frac{1}{r})}\|u(t)\|_{sr} \leqslant 2R,\right.$$
$$\left. t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|u(t)\|_{sq} \leqslant b, \forall t \in (0,T), \lim_{t\to 0^+} t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|u(t)\|_{sq} = 0\right\}.$$

We estimate as before and remark that

$$\lim_{t\to 0^+} t^{\frac{N}{2s}\left(1-\frac{1}{q}\right)}\|K(t) * u_0\|_{sq} = 0$$

we can take appropriate T, b such that Φ is a contraction on E. □

The remainder of the proof is similar to that of theorem 2.1 of [8], so we omit it.

3. Global solutions

Lemma 1 [8, lemma 3.1; 14, proposition 1]. *Let* $v_0 \in L^1(R^N) \cap L^m(R^N)(1 \leqslant m \leqslant \infty)$ *be a non-negative function,* $v_0 \not\equiv 0$, $\lambda > 0$ *and* $q \geqslant 1$. *Then there exists a unique, positive solution* v *of*

$$v_t - \Delta v = (1+t)^{\lambda} \boldsymbol{a} \cdot \nabla (v^q) \tag{10}$$

such that $v(0, x) = v_0(x)$,

$$\begin{aligned} &v \in C((0,\infty); W^{2,m}(R^N)) \cap C^1((0,\infty); L^m(R^N)) \cap C([0,\infty); L^m(R^N)), m \in [1,\infty] \\ &\|v(t)\|_m \leqslant C_0 \|v_0\|_1 (1+t)^{-\frac{N}{2}(1-\frac{1}{m})} \qquad m \in [1,\infty) \end{aligned} \tag{11}$$

for a constant C_0 *and*

$$\|v(t)\|_\infty \leqslant C_1(\|v_0\|_1 + \|v_0\|_\infty)(1+t)^{-N/2} + C_2 \|v_0\|_1^q (1+t)^{\lambda + \frac{1-Nq}{2}} \tag{12}$$

for some constants C_1 *and* C_2. *If moreover* $1 < q < 2$, *then there exists a constant* C_3 *such that*

$$\|v(t)\|_\infty \leqslant C_3(\|v_0\|_1 + \|v_0\|_\infty)(1+t)^{-\frac{N+1}{2q} - \frac{\lambda}{q}}. \tag{13}$$

Lemma 2. *Let* $u(t,x), v(t,x) \in C^1((0,T) \times R^N)$ *be non-negative functions,* $u(t,\cdot), v(t,\cdot) \in H^2(R^N) \cap L^\sigma(R^N) \cap L^r(R^N) \cap L^\infty(R^N)$, $\Delta u(t,\cdot), \Delta v(t,\cdot) \in L^1(R^N)(0 < t < T)$. *If*

$$\begin{cases} u_t - \Delta u \geqslant \left(\int_{R^N} u(t,x)^\sigma \,\mathrm{d}x\right)^{p/\sigma} u^r + \boldsymbol{a} \cdot \nabla (u^q) & t > 0, \quad x \in R^N \\ v_t - \Delta v \leqslant \left(\int_{R^N} v(t,x)^\sigma \,\mathrm{d}x\right)^{p/\sigma} v^r + \boldsymbol{a} \cdot \nabla (v^q) & t > 0, \quad x \in R^N \\ u(0,x) \geqslant v(0,x) & x \in R^N \end{cases}$$

then for all $(t,x) \in (0,T) \times R^N$, *we have*

$$u(t,x) \geqslant v(t,x).$$

Proof. Subtracting the inequalities satisfied by u, v, we get

$$(v-u)_t - \Delta(v-u) \leqslant \|v\|_\sigma^p v^r - \|u\|_\sigma^p u^r + \boldsymbol{a} \cdot \nabla (v^q - u^q).$$

Let $\Omega(t) = \{x \in R^N : v(t,x) > u(t,x)\}$. We have from an argument in [8, lemma 2.2]

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega(t)} (v-u)\,\mathrm{d}x &\leqslant \int_{\Omega(t)} \left[\|v\|_\sigma^p v^r - \|u\|_\sigma^p u^r\right] \mathrm{d}x \\ &= \int_{\Omega(t)} \left[\|v\|_\sigma^p (v^r - u^r) + u^r \left(\|v\|_\sigma^p - \|u\|_\sigma^p\right)\right] \mathrm{d}x. \end{aligned}$$

Since

$$\begin{aligned} \|v\|_\sigma^p - \|u\|_\sigma^p &\leqslant \frac{p}{\sigma} \|\theta_1 v + (1-\theta_1)u\|_\sigma^{p-\sigma} \sigma \int_{R^N} [\theta_1 v + (1-\theta_1)u]^{\sigma-1} (v-u)\,\mathrm{d}x \\ &\leqslant p \|\theta_1 v + (1-\theta_1)u\|_\sigma^{p-\sigma} \|\theta_1 v + (1-\theta_1)u\|_\infty^{\sigma-1} \int_{\Omega(t)} (v-u)\,\mathrm{d}x \end{aligned}$$

where $\theta_1 = \theta_1(s) \in (0,1)$, then

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega(t)} (v-u)\,\mathrm{d}x &\leqslant \|v\|_\sigma^p \int_{\Omega(t)} r[\theta_2 v + (1-\theta_2)u]^{r-1} (v-u)\,\mathrm{d}x + p\|\theta_1 v \\ &\quad + (1-\theta_1)u\|_\sigma^{p-\sigma} \|\theta_1 v + (1-\theta_1)u\|_\infty^{\sigma-1} \int_{\Omega(t)} (v-u)\,\mathrm{d}x \int_{\Omega(t)} u^r\,\mathrm{d}x \\ &\leqslant C \int_{\Omega(t)} (v-u)\,\mathrm{d}x. \end{aligned}$$

where $\theta_2 = \theta_2(s) \in (0, 1)$, $C = r\|v\|_\sigma^p(\|v\|_\infty + \|u\|_\infty)^{r-1} + p\|u\|_r^r(\|v\|_\sigma + \|u\|_\sigma)^{p-\sigma}(\|v\|_\infty + \|u\|_\infty)^{\sigma-1}$. We remark that for $t = 0$, $\int_{\Omega(t)}(v-u)\,\mathrm{d}x = 0$, by applying Gronwall's inequality, we obtain

$$\int_{\Omega(t)} (v-u)\,\mathrm{d}x = 0 \qquad 0 < t < T.$$

By the continuity of u, v we get the result of this lemma. □

We are now ready to state the theorem on the existence of global solutions.

Theorem 3. *Suppose for $r = 1$, $p > 2/N + p/\sigma$ and $q \geqslant 1$, for $r > 1$, $p + r > 1 + 2/N + p/\sigma$ and $q \geqslant 1$; or $1 + p/\sigma + (3-r)N < p + r \leqslant 1 + 2/N + p/\sigma$ and $1 < q < (N+1)(r-1)/[2 - Np(1-1/\sigma)]$. There exists a constant $C > 0$ such that if $u_0 \in L^1 \cap L^\infty \cap L^\sigma$ is a non-negative function satisfying*

$$\|u_0\|_1 + \|u_0\|_\infty + \|u_0\|_\sigma \leqslant C$$

then the unique solution of (1) is global.

Proof. Let $U(t,x) = (1+t)^\lambda v$ where $\lambda > 0$ will be fixed later and v is a solution of

$$v_t - \Delta v = (1+t)^{\lambda(q-1)}\boldsymbol{a}\cdot \bigtriangledown (v^q) \qquad v(0,x) = u_0(x). \tag{14}$$

We shall prove that if $\|u_0\|_1 + \|u_0\|_\infty + \|u_0\|_\sigma$ is small enough, then U is a supersolution of (1). We have

$$U_t - \Delta U = \boldsymbol{a}\cdot \bigtriangledown (U^q) + \lambda(1+t)^{\lambda-1}v.$$

Therefore, it will be enough to show that

$$\lambda(1+t)^{\lambda-1}v \geqslant (1+t)^{\lambda(p+r)}\left(\int_{R^N} v^\sigma(t,x)\,\mathrm{d}x\right)^{p/\sigma} v^r. \tag{15}$$

For $r = 1$, we know from (15) that if

$$\left(\int_{R^N} v^\sigma(t,x)\,\mathrm{d}x\right)^{p/\sigma} \leqslant \lambda(1+t)^{-1-\lambda p}$$

then U is a supersolution of (1). From (11), we have

$$\left(\int_{R^N} v^\sigma(t,x)\,\mathrm{d}x\right)^{p/\sigma} \leqslant \left[C_0\|u_0\|_1(1+t)^{-N(1-1/\sigma)/2}\right]^p.$$

Set

$$\left[C_0\|u_0\|_1(1+t)^{-N(1-1/\sigma)/2}\right]^p \leqslant \lambda(1+t)^{-1-\lambda p}. \tag{16}$$

From the condition

$$\frac{2}{N} + \frac{p}{\sigma} < p$$

we can choose $\lambda > 0$ such that

$$\frac{Np}{2}\left(1 - \frac{1}{\sigma}\right) > 1 + \lambda p.$$

Then (16) holds for some $\lambda > 0$ and t large.

For t in the neighbourhood of zero and the $\lambda > 0$ taken previously, it is enough to choose $\|u_0\|_1 + \|u_0\|_\infty + \|u_0\|_\sigma$ small (then $C_0\|u_0\|_1$ small) such that (16) holds.

For $r > 1$, from (11) we know that if

$$\lambda(1+t)^{\lambda-1}v \geqslant (1+t)^{\lambda(p+r)}\left[C_0\|u_0\|_1(1+t)^{-N(1-1/\sigma)/2}\right]^p v^r$$

or equivalently

$$v \leqslant \lambda^{1/(r-1)}(C_0\|u_0\|_1)^{-p/(r-1)}(1+t)^{[Np(1-1/\sigma)/2-\lambda(p+r-1)-1]/(r-1)} \tag{17}$$

then (15) holds.

We now distinguish two different cases: $1 < q < 1 + 1/N$ and $1 + 1/N \leqslant q$.

(i) First suppose that $1 + 1/N \leqslant q$. From (12) we have

$$v \leqslant C_1(\|u_0\|_1 + \|u_0\|_\infty)(1+t)^{-N/2} + C_2\|u_0\|_1(1+t)^{\lambda(q-1)+(1-Nq)/2}.$$

So, if we can choose $\lambda > 0$ such that the right-hand side of the last inequality is bounded by

$$\lambda^{1/(r-1)}(C_0\|u_0\|_1)^{-p/(r-1)}(1+t)^{[Np(1-1/\sigma)/2-\lambda(p+r-1)-1]/(r-1)}$$

then U is a supersolution of (1). For $t > 0$ large enough, we set $\lambda > 0$ satisfying

$$-\frac{1}{r-1}\left[\frac{Np}{2}\left(1-\frac{1}{\sigma}\right) - \lambda(p+r-1) - 1\right] \leqslant \min\left\{\frac{N}{2}, -\lambda(q-1) + \frac{Nq-1}{2}\right\}$$

or equivalently

$$\begin{aligned} 0 < \lambda \leqslant \min\Bigg\{ & \frac{r-1}{p+r-1}\left(\frac{N}{2} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1}\right) \\ & \frac{r-1}{q(r-1)+p}\left(\frac{Nq-1}{2} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1}\right)\Bigg\}. \end{aligned} \tag{18}$$

We remark that $1 + 1/N \leqslant q$, that is, $(Nq-1)/2 \geqslant N/2$, and we have

$$\frac{Nq-1}{2} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1} \geqslant \frac{N}{2} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1}.$$

On the one hand, the condition

$$p + r > 1 + \frac{2}{N} + \frac{p}{\sigma} \qquad r > 1$$

implies

$$\frac{N}{2} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1} > 0.$$

On the other hand, if

$$\begin{aligned} &\frac{r-1}{q(r-1)+p}\left(\frac{Nq-1}{2} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1}\right) \\ &\qquad < \frac{r-1}{p+r-1}\left(\frac{N}{2} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1}\right) \end{aligned}$$

then we can prove that

$$q < \frac{\sigma(p+r+1) + Np}{2\sigma + Np}.$$

We remark that $q \geqslant 1 + 1/N$, then we have

$$\frac{\sigma(p+r+1) + Np}{2\sigma + Np} > 1 + \frac{1}{N}.$$

Furthermore, we have

$$p + r > 1 + \frac{2}{N} + \frac{p}{\sigma}.$$

Therefore, using the conditions of our theorem, we confirm that when $1 + 1/N \leqslant q$, the case $p + r \leqslant 2/N + p/\sigma + 1$ will not occur. So in the range of p, q, r, σ given by the theorem, the right-hand side of (18) is strictly positive. Thus there exists $\lambda > 0$ satisfying (18).

For t in the neighbourhood of zero and the $\lambda > 0$ taken previously, it is enough to choose $\|u_0\|_1 + \|u_0\|_\infty + \|u_0\|_\sigma$ small such that (17) holds.

(ii) Now suppose $1 < q < 1 + 1/N$. Since $1 < q < 2$, from (13) we have

$$v \leqslant C_3(\|v_0\|_1 + \|v_0\|_\infty)(1+t)^{-\frac{1+\lambda(q-1)}{q} - \frac{N-1}{2q}}. \tag{19}$$

Arguing as before, we shall prove that U is a supersolution of (1). We first choose $\lambda > 0$ such that

$$-\frac{1}{r-1}\left[\frac{Np}{2}\left(1-\frac{1}{\sigma}\right) - \lambda(p+r-1) - 1\right] \leqslant \frac{1+\lambda(q-1)}{q} + \frac{N-1}{2q}$$

or equivalently

$$0 < \lambda \leqslant \frac{q(r-1)}{pq+r-1}\left[\frac{N+1}{2q} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1}\right]. \tag{20}$$

Since $1 < q < 1+1/N$, it is easy to see that $(N+1)/2q > N/2$. So, if $p+r > 1+2/N+p/\sigma$, or if $1+p/\sigma+(3-r)/N < p+r \leqslant 1+2/N+p/\sigma$ and $1 < q < (N+1)(r-1)/[2-Np(1-1/\sigma)]$, then

$$\frac{N+1}{2q} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1} > 0.$$

Therefore, there exists λ satisfying (20), that is, (17) holds for some $\lambda > 0$ and t large.

For t in the neighbourhood of zero and the $\lambda > 0$ taken previously, it is enough to choose $\|u_0\|_1 + \|u_0\|_\infty + \|u_0\|_\sigma$ small such that (17) holds. The proof is complete. □

Remark 1. Let

$$\frac{N+1}{2q} + \frac{Np}{2(r-1)}\left(1-\frac{1}{\sigma}\right) - \frac{1}{r-1} > 0$$

we have

$$\begin{aligned} p+r &> 1 + \frac{p}{\sigma} + \frac{2}{N}\left[1 + \frac{(Nq-N-1)(r-1)}{2q}\right] \\ &= 1 + \frac{p}{\sigma} + \frac{2}{N} + \frac{(Nq-N-1)(r-1)}{Nq}. \end{aligned}$$

We remark that the function

$$h(q,r) = \frac{(Nq-N-1)(r-1)}{Nq}$$

satisfies $h(q,1) = 0$ and $\partial h/\partial q > 0$, $(r > 1, q > 1)$, thus

$$h(q,r) > h(1,r) = \frac{1-r}{N}.$$

We recall that when $r > 1, q \geqslant 1 + 1/N$ we have $\min\{0, (Nq-N-1)(r-1)/(Nq)\} = 0$, and when $r > 1, 1 < q < 1 + 1/N$ we have $\min\{0, (Nq-N-1)(r-1)/(Nq)\} = (Nq-N-1)(r-1)/(Nq)$; therefore, theorem 3 can also be stated as the following theorem.

Theorem 3′. *Suppose $q > 1$ and*

$$p + r > 1 + \frac{p}{\sigma} + \frac{2}{N} + \min\left\{0, \frac{(Nq-N-1)(r-1)}{Nq}\right\}. \tag{21}$$

There exists a constant $C > 0$ such that if $u_0 \in L^1 \cap L^\infty \cap L^\sigma$ is a non-negative function satisfying

$$\|u_0\|_1 + \|u_0\|_\infty + \|u_0\|_\sigma \leqslant C$$

then the unique solution of (1) is global.

Remark 2. If $p = 0$, then the results of theorem 3′ coincide with that of theorem A.

4. Blow-up

In this section, we state and prove the blow-up results announced in the introduction for positive solutions of (1). Throughout, u will denote a positive, regular solution of (1) in $[0, T) \times R^N$ with initial value u_0 non-negative and not identically zero. By regularity [8, 15, 16], we may assume that u_0 is a smooth function and for a constant $\alpha > 0$, $0 \leqslant u_0(x) \leqslant \|u_0\|_\infty \mathrm{e}^{-\alpha|x|^2}$. Also, we assume that $q \geqslant 1$, since the case $q = 1$ can be reduced to the known results for (7) by a simple change of variables.

First we give the following lemma which is the so-called inverse form of Hölder inequality.

Lemma 3 [17]. *Suppose* $0 < \theta < 1$, $\theta' = \theta/(\theta - 1)$, $f \in L^\theta(R^N)$, $0 < \int_{R^N} |g(x)|^{\theta'}\,\mathrm{d}x < \infty$. *Then*

$$\int_{R^N} |f(x)g(x)|\,\mathrm{d}x \geqslant \left(\int_{R^N} |f(x)|^\theta\,\mathrm{d}x\right)^{1/\theta} \left(\int_{R^N} |g(x)|^\theta\,\mathrm{d}x\right)^{1/\theta'}. \tag{22}$$

Lemma 4 [8, Lemma 4.1] . *Let f and g be positive C^1 real functions defined on $[0, T)$ such that*

$$f'(t) \geqslant C[g^p(t) - F(g(t))] \qquad f(t) \leqslant g(t) \qquad t \in [0, T)$$

where $p > 1$, C is a positive constant and F is a continuous function defined on $[0, \infty)$ that satisfies the following conditions:

(1) $r^{-p}F(r)$ is decreasing;

(2) there exists $r_0 > 0$ such that $r^p - F(r) > 0$ for all $r > r_0$. Then $T < \infty$ whenever $f(0) > r_0$.

Theorem 4. *Let $1 < q \leqslant \sigma(p + r)/(p + \sigma)$. Then for some $\lambda > 0$ ($0 < \lambda < 1/(2K|\boldsymbol{a}|)$ if $p = 0$, $1 < q = r$)*

$$\int_{R^N} u_0(x)\phi(\lambda x)\,\mathrm{d}x > \begin{cases} \max\left\{2^{\frac{1}{p+r-1}}\lambda^{\frac{2}{p+r-1}-N}, (2K|\boldsymbol{a}|)^{\frac{1}{p+r-q}}\lambda^{\frac{1}{p+r-q}-N}\right\} \\ \qquad p > 0 \quad 1 < q \leqslant \frac{\sigma(p+r)}{p+\sigma} \\ \max\left\{2^{\frac{1}{r-1}}\lambda^{\frac{2}{r-1}-N}, (2K|\boldsymbol{a}|)^{\frac{1}{r-q}}\lambda^{\frac{1}{r-q}-N}\right\} & p = 0 \quad 1 < q < r \\ 2^{\frac{1}{r-1}}\lambda^{\frac{2}{r-1}-N} \qquad p = 0 \quad 1 < q = r \end{cases} \tag{23}$$

then u blows up in finite time, where $\phi(x)$ is given by (5).

Proof. Suppose u is a global solution of (1). Let us define for $\lambda > 0$

$$\phi_\lambda(x) = \lambda^N \phi(\lambda x)$$

and

$$f(t) = \int_{R^N} u(t, x)\phi_\lambda(x)\,\mathrm{d}x \qquad g(t) = \left(\int_{R^N} u^{\sigma(p+r)/(p+\sigma)}(t, x)\phi_\lambda(x)\,\mathrm{d}x\right)^{(p+\sigma)/[\sigma(p+r)]}.$$

Then

$$\int_{R^N} \phi_\lambda(x)\,\mathrm{d}x = 1 \qquad \Delta\phi_\lambda(x) \geqslant -\lambda^2\phi_\lambda(x) \qquad |\nabla\phi_\lambda(x)| \leqslant K\lambda\phi_\lambda(x) \tag{24}$$

and, from Hölder's inequality

$$f(t) \leqslant g(t).$$

Multiplying the equation in (1) by $\phi_\lambda(x)$, integrating over R^N and using (24), we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_{R^N} u\phi_\lambda\,\mathrm{d}x &= \int_{R^N}\Delta u\phi_\lambda\,\mathrm{d}x + \int_{R^N}\left(\int_{R^N} u^\sigma\,\mathrm{d}x\right)^{p/\sigma} u^r\phi_\lambda\,\mathrm{d}x + \int_{R^N}\boldsymbol{a}\cdot\nabla(u^q)\phi_\lambda\,\mathrm{d}x\\ &= \int_{R^N}\Delta u\phi_\lambda\,\mathrm{d}x + \left(\int_{R^N} u^\sigma\,\mathrm{d}x\right)^{p/\sigma}\int_{R^N} u^r\phi_\lambda\,\mathrm{d}x + \int_{R^N} u^a\boldsymbol{a}\cdot\nabla\phi_\lambda\,\mathrm{d}x\\ &\geqslant \left(\int_{R^N} u^\sigma\phi_\lambda\,\mathrm{d}x\right)^{p/\sigma}\int_{R^N} u^r\phi_\lambda\,\mathrm{d}x - \lambda^2\int_{R^N} u\phi_\lambda\,\mathrm{d}x - \lambda K|\boldsymbol{a}|\int_{R^N} u^q\phi_\lambda\,\mathrm{d}x.\end{aligned} \tag{25}$$

In lemma 3, we take

$$f(x) = u^{\sigma-s}\phi_\lambda^{1/\theta} \qquad g(x) = u^s\phi_\lambda^{1/\theta'}$$

where $\theta = p/(p+\sigma)$, $\theta' = -p/\sigma$, $s = -\sigma r/p$. From the result of lemma 3, we have

$$\begin{aligned}\left(\int_{R^N} u^\sigma\phi_\lambda\,\mathrm{d}x\right)^{p/\sigma} &= \left(\int_{R^N}\left(u^{\sigma-s}\phi_\lambda^{1/\theta}\right)\left(u^s\phi_\lambda^{1/\theta'}\right)\mathrm{d}x\right)^{p/\sigma}\\ &\geqslant \left(\int_{R^N} u^{\theta(\sigma-s)}\phi_\lambda\,\mathrm{d}x\right)^{p/\sigma\theta}\left(\int_{R^N} u^{\theta' s}\phi_\lambda\,\mathrm{d}x\right)^{p/\sigma\theta'}\\ &= \left(\int_{R^N} u^{\sigma(p+r)/(p+\sigma)}\phi_\lambda\,\mathrm{d}x\right)^{(p+\sigma)/\sigma}\left(\int_{R^N} u^r\phi_\lambda\,\mathrm{d}x\right)^{-1}.\end{aligned}$$

Substituting the last inequality into (25) we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{R^N} u\phi_\lambda\,\mathrm{d}x \geqslant \left(\int_{R^N} u^{\sigma(p+r)/(p+\sigma)}\phi_\lambda\,\mathrm{d}x\right)^{(p+\sigma)/\sigma} - \lambda^2\int_{R^N} u\phi_\lambda\,\mathrm{d}x - \lambda K|\boldsymbol{a}|\int_{R^N} u^q\phi_\lambda\,\mathrm{d}x. \tag{26}$$

First we suppose $p > 0$, $1 < q \leqslant \sigma(p+r)/(p+\sigma)$. We remark that $q < p+r$, using Hölder's inequality in (26), we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_{R^N} u\phi_\lambda\,\mathrm{d}x \geqslant{}& \left(\int_{R^N} u^{\frac{\sigma(p+r)}{p+\sigma}}\phi_\lambda\mathrm{d}x\right)^{\frac{p+\sigma}{\sigma}} - \lambda^2\int_{R^N} u\phi_\lambda\,\mathrm{d}x - \lambda K|\boldsymbol{a}|\\ &\times\left(\int_{R^N} u^{\frac{\sigma(p+r)}{p+\sigma}}\phi_\lambda\,\mathrm{d}x\right)^{q(p+\sigma)/[\sigma(p+r)]}.\end{aligned}$$

That is,

$$\begin{aligned}f'(t) &\geqslant g^{p+r}(t) - \lambda^2 f(t) - \lambda K|\boldsymbol{a}|g^q(t)\\ &\geqslant g^{p+r}(t) - \lambda^2 g(t) - \lambda K|\boldsymbol{a}|g^q(t).\end{aligned}$$

We remark that $f(0) = \int_{R^N} u_0(x)\phi_\lambda(x)\,\mathrm{d}x > 0$, and from the proof of lemma 4 (cf [8, lemma 4.1]) we know that if

$$f^{p+r}(0) - (\lambda^2 f(0) + \lambda K|\boldsymbol{a}|f^q(0)) > 0 \tag{27}$$

then f blows up in finite time. The last condition is satisfied if

$$\int_{R^N} u_0(x)\phi_\lambda(x)\,\mathrm{d}x > \max\left\{2^{\frac{1}{p+r-1}}\lambda^{\frac{2}{p+r-1}}, (2K|\boldsymbol{a}|)^{\frac{1}{p+r-q}}\lambda^{\frac{1}{p+r-q}}\right\}$$

or equivalently if

$$\int_{R^N} u_0(x)\phi(\lambda x)\,\mathrm{d}x > \max\left\{2^{\frac{1}{p+r-1}}\lambda^{\frac{2}{p+r-1}-N}, (2K|\boldsymbol{a}|)^{\frac{1}{p+r-q}}\lambda^{\frac{1}{p+r-q}-N}\right\}. \tag{28}$$

Hence, u cannot be global, thus proving the theorem when $p > 0$ and $1 < q \leqslant \sigma(p+r)/(p+\sigma)$.

Now suppose that $p = 0$. Problem (1) is equivalent to that studied in [8]. From theorem 4.2 of [8], if for some $\lambda > 0$ $(0 < \lambda < 1/(2K|\boldsymbol{a}|)$ if $p = 0, 1 < q = r)$

$$\int_{R^N} u_0(x)\phi(\lambda x)\,\mathrm{d}x > \begin{cases} \max\left\{2^{\frac{1}{r-1}}\lambda^{\frac{2}{r-1}-N}, (2K|\boldsymbol{a}|)^{\frac{1}{r-q}}\lambda^{\frac{1}{r-q}-N}\right\} & p = 0,\ 1 < q < r \\ 2^{\frac{1}{r-1}}\lambda^{\frac{2}{r-1}-N} & p = 0,\ 1 < q = r. \end{cases} \tag{29}$$

then u blows up in finite time, where $\phi(x)$ is given by (5). Using (28) and (29) we complete the proof. □

Corollary 1. *If*

$$p > 0 \quad r > 1 \quad \sigma \geqslant 1 \quad p \neq \sigma \quad 1 + 1/N \leqslant q \leqslant \sigma(p+r)/(p+\sigma) \quad p + r < 1 + 2/N$$

then (1) cannot have nontrivial non-negative global solutions.

Proof. *We remark that $p + r < 1 + 2/N \leqslant q + 1/N$, let $\lambda \to 0^+$ in (28), the left-hand side converges to* □

$$\phi(0)\int_{R^N} u_0(x)\,\mathrm{d}x > 0$$

while the right-hand side converges to zero. Therefore, we can take $\lambda > 0$ so small that condition (28) holds, completing the proof.

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